Graphical Representation of Causal Effects

November 10, 2016

Lord's Paradox: Observed Data

	Covariates (X)	June weight		
Students	Sex, Sept. weight	Y(0)	Y(1)	Impact
1	X_1	?	$Y_1(1)$	?
2	X_2	?	$Y_2(1)$	?
3	X_3	?	$Y_3(1)$	?
$\vdots$	$\vdots$	$\vdots$		
N	X_N	?	$Y_N(1)$	?

Units: Students; Covariates: Sex, September Weight;
Potential Outcomes: June Weight under Treatment and Control;
Treatment = University diet; Control = ??

Statistician 1: June weight under control = September weight
Statistician 2: June weight under control = a linear function of September weight, i.e.

$$E[Y(0)] = \beta_0 + \beta_1 Sex + \beta_2 Weight_{sep}$$

Wainer H and Brown L (2007). Three Statistical Paradoxes in the Interpretation of Group Differences: Illustrated with Medical School Admission and Licsencing Data. *Handbook of Statistics*.

Assignment Mechanism

- Determines which units receive treatment, which receive control
- $P(\, T \mid X, Y(0), Y(1)\,)$
- Known for randomized trials; unknown for observational studies
- Model for assignment mechanism necessary (sometimes sufficient)
  - Model of "science", $P(\, Y(0), Y(1) \mid X\,)$ not necessary if one knows the assignment mechanism, e.g., randomized trials
- So, what's wrong with the assignment mechanism in Lord's Paradox?

Key Property of Randomized Trials

- Treatment assignment is "unconfounded", also known as "conditional exchangeability"
  - $P(T \mid X, Y(0), Y(1)) = P(T \mid X)$
  - Assignment does not depend on potential outcomes
  - Removes confounding of all variables
  - Crucial for observational studies, but usually as an unverifiable assumption
- Positivity: each unit has a positive probability of receiving each treatment
  - $0 < P(T \mid X) < 1$ for all X
  - Everyone in the study relevant for comparisons
- Study must be designed without the use of the knowledge of outcomes

Randomization ensures balance of covariates.

Example: Truth vs Observation

	A	Y	Y^0	Y^1
Rheia	0	0	0	?
Kronos	0	1	1	?
Demeter	0	0	0	?
Hades	0	0	0	?
Hestia	1	0	?	0
Poseidon	1	0	?	0
Hera	1	0	?	0
Zeus	1	1	?	1
Artemis	0	1	1	?
Apollo	0	1	1	?
Leto	0	0	0	?
Ares	1	1	?	1
Athena	1	1	?	1
Hephaestus	1	1	?	1
Aphrodite	1	1	?	1
Cyclope	1	1	?	1
Persephone	1	1	?	1
Hermes	1	0	?	0
Hebe	1	0	?	0
Dionysus	1	0	?	0

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Causal Diagram

- Directed Acyclic Graph vs *Causal* Directed Acyclic Graph

- Can represent both association and causation
- Absence of an arrow from A to Y means no individual in the population has that direct causal effect; Presence of an arrow from A to Y means there is at least one individual in the population having the causal effect
- All common causes, even if unmeasured, of any pair of variables on the graph are themselves on the graph
- Any Variable is a cause of its descendants

Causal Diagram (continued)

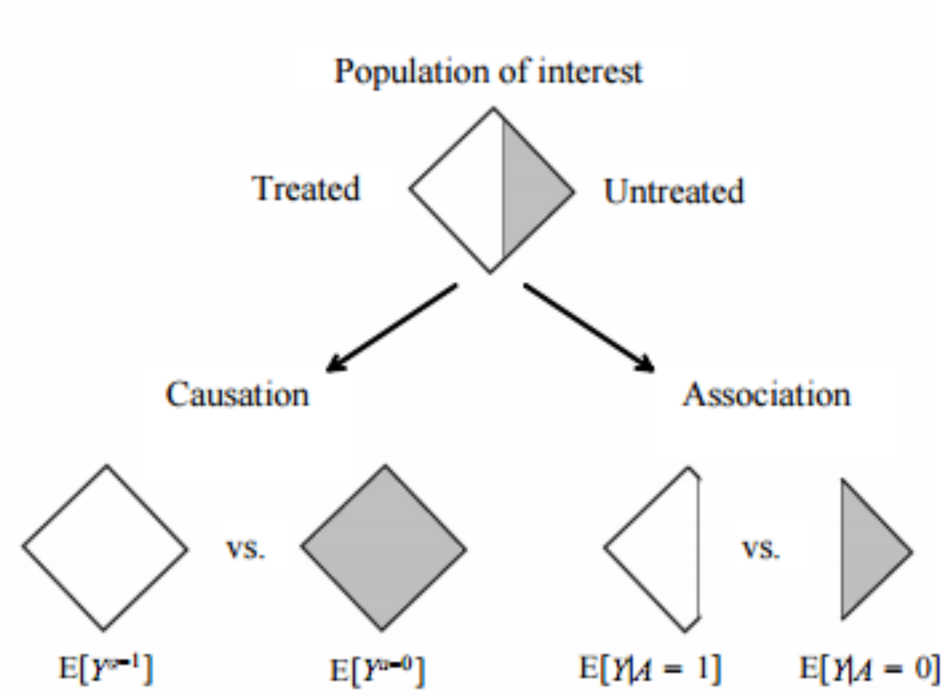
- A standard causal diagram does not distinguish whether an arrow represent a harmful effect or protective effect
- A variable, if having two causes, the diagram does not encode how the two causes inter

Causal Markov Assumption

- Causal DAGs are of no practical use unless we make an assumption linking the causal structure represented by the DAG to the data obtained in a study. We refer to such assumptions as **causal Markov assumption**:

- Conditional on its direct causes, a variable is independent of any variable for which it is not a cause
- Equivalent to: conditional on its parents, a node is independent of its non-descendants
- Mathematically equivalent to the statement that the density $f(V)$ of all the variables *V* in DAG *G* satisfies the Markov factorization $f(v) = \prod_{j=1}^{M} f(v_j \mid Pa_j)$

Association vs Causation



Causal Diagram for Structural Representation of Biases under the Null

- Common causes for treatment A and outcome Y

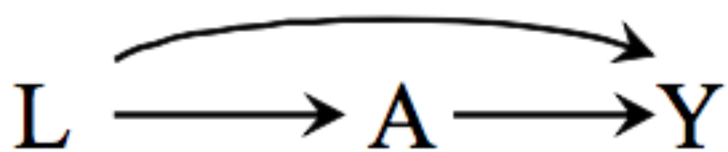
- Common effect for treatment A and outcome Y

- Measurement error on the nodes

Assignment Mechanism

- Marginal Randomization

- Conditional Randomization



- Can the above represent observational studies? (Equivelent to assuming conditional exchangeability)

Exchangeability

- Unconditional Exchangeability

- Conditional Exchangeability

Stratum M=1

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

Effect Modification and Cancellation of Effects

	M	Y^0	Y^1
Rheia	1	0	1
Demeter	1	0	0
Hestia	1	0	0
Hera	1	0	0
Artemis	1	1	1
Leto	1	0	1
Athena	1	1	1
Aphrodite	1	0	1
Persephone	1	1	1
Hebe	1	1	0
Kronos	0	1	0
Hades	0	0	0
Poseidon	0	1	0
Zeus	0	0	1
Apollo	0	1	0
Ares	0	1	1
Hephaestus	0	0	1
Cyclope	0	0	1
Hermes	0	1	0
Dionysus	0	1	0

Effect Modification Under Conditional Randomization or Conditional Exchangeability

Stratum $M = 0$

	L	A	Y
Cybele	0	0	0
Saturn	0	0	1
Ceres	0	0	0
Pluto	0	0	0
Vesta	0	1	0
Neptune	0	1	0
Juno	0	1	1
Jupiter	0	1	1
Diana	1	0	0
Phoebus	1	0	1
Latona	1	0	0
Mars	1	1	1
Minerva	1	1	1
Vulcan	1	1	1
Venus	1	1	1
Seneca	1	1	1
Proserpina	1	1	1
Mercury	1	1	0
Juventas	1	1	0
Bacchus	1	1	0

Stratum M=1

	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
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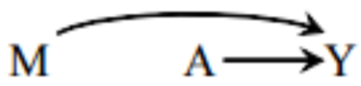
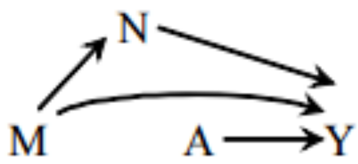
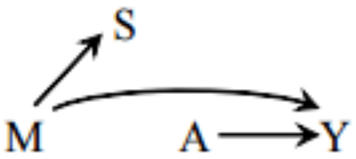
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

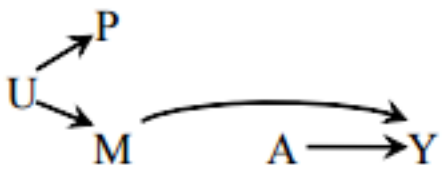
Causal Diagram for Effect Modification (with causal effect on outcome)

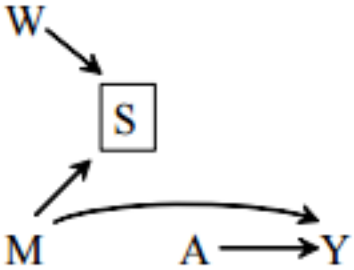






Causal Diagram for Effect Modification (without causal effect on outcome)





Alternative Representations

- **Single World Intervention Graph (SWIG, Richardson and Robins, 2013):** seamlessly unifies the counterfactual and graphical approaches to causality by explicitly including the counterfactual variables on the graph

- **Influence Diagrams**. Based on decision theory (Dawid, 2000, 2002). Make no reference to counterfactuals and uses causal diagrams augmented with decision nodes to represent the interventions of interest.

Reading

- Hernan and Robins (2016), Chapter 6, Causal Inference. https://www.hsph.harvard.edu/miguel-hernan/causal-inference-book/