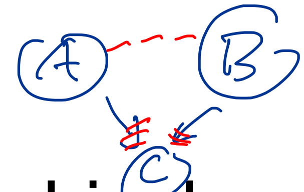
Junction Tree Algorithm Examples

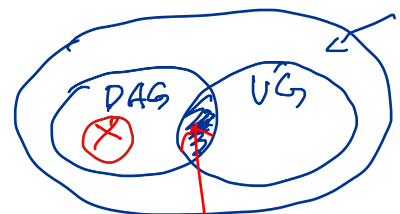
October 13, 2016

Junction Tree Algorithm



- Moralize (if starting from a directed graphical model) ✓

all distri



- Triangulate (make it chordal) ✓
- Construct a junction tree (maximum cardinality search)

have

[Theorem: Chordal graph $\Leftrightarrow$ existence of a junction tree representation]

- Define potentials on maximal cliques
- Introduce evidence (if any)
- Propagate probabilities

Lauritzen et al (1990) Network

$$Pr(X_j \mid X_{-j}) = Pr(X_j \mid N(X_j))$$

read this from an UG

CHILD Example from Spiegelhalter et al (1993) Statistical Science

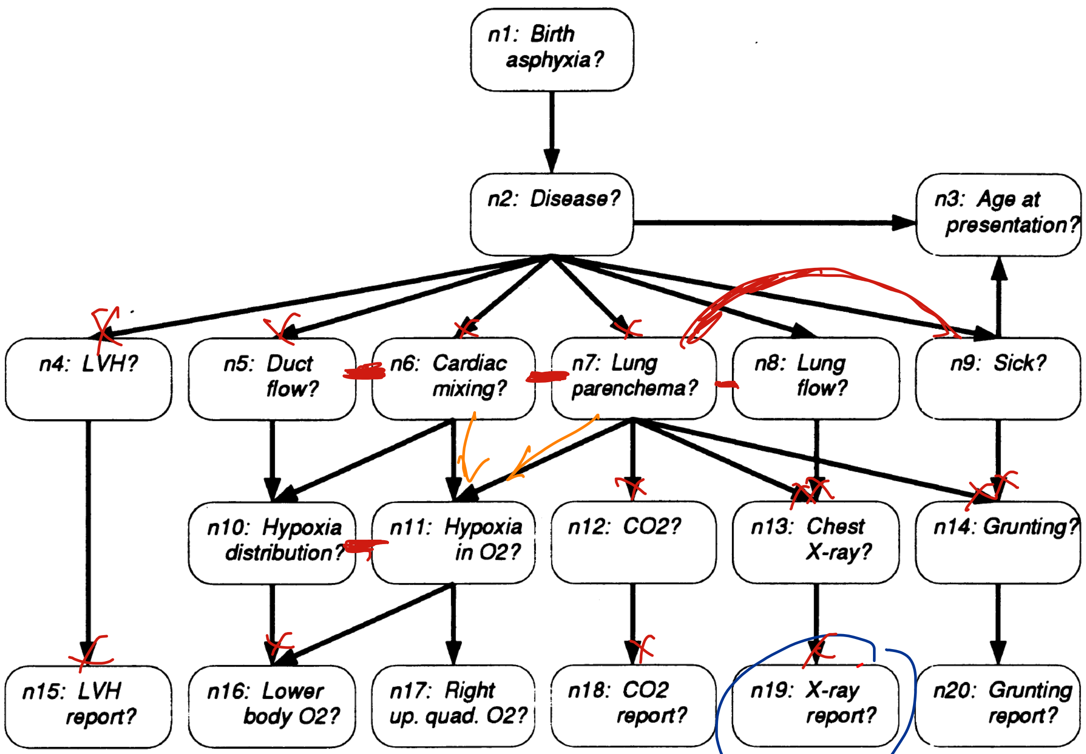


FIG. 2. *Directed acyclic graph representing the incidence and presentation of six possible diseases that would lead to a "blue" baby. LVH, left ventricular hypertrophy.*

Conditional Probability Tables

TABLE 1

Subjective assessments of conditional probability tables assessed by expert for links $n2 \rightarrow n4$ and $n4 \rightarrow n15$

n2: Disease?	*n4: LVH?* Yes	+	*n4: LVH?* No	
PFC	0.10	+	0.90	=1
TGA	0.10		0.90	=1
Fallot	0.10		0.90	
PAIVS	0.90		0.10	
TAPVD	0.05		0.95	
Lung	0.10		0.90	=1

sensitivity

n4: LVH?	*n15: LVH-report?* Yes	+	*n15: LVH-report?* No	
Yes	0.90	+	0.10	=1
No	0.05		0.95	

specificity.

Visualization of Updated Beliefs on Every Node

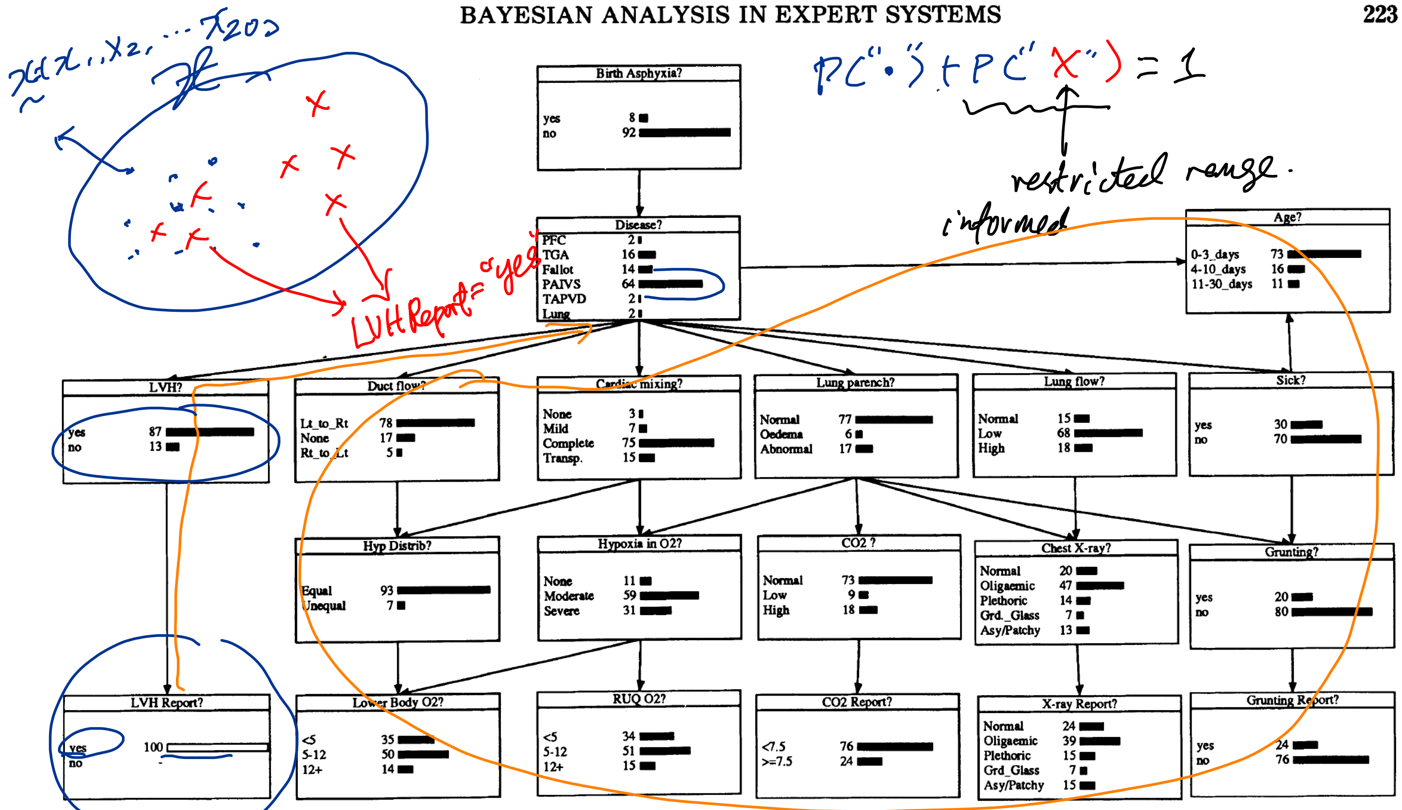


FIG. 3. *Conditional probability distributions on all nodes after propagation of evidence LVH-report = yes. The numbers and the length of the bars represent the current probability: for example, 64% belief that PAIVS is the true diagnosis, compared to a prior 22% belief. For observed evidence, that is, LVH-report = yes, the bar is hollow.*

Visualization of Updated Beliefs on Every Node

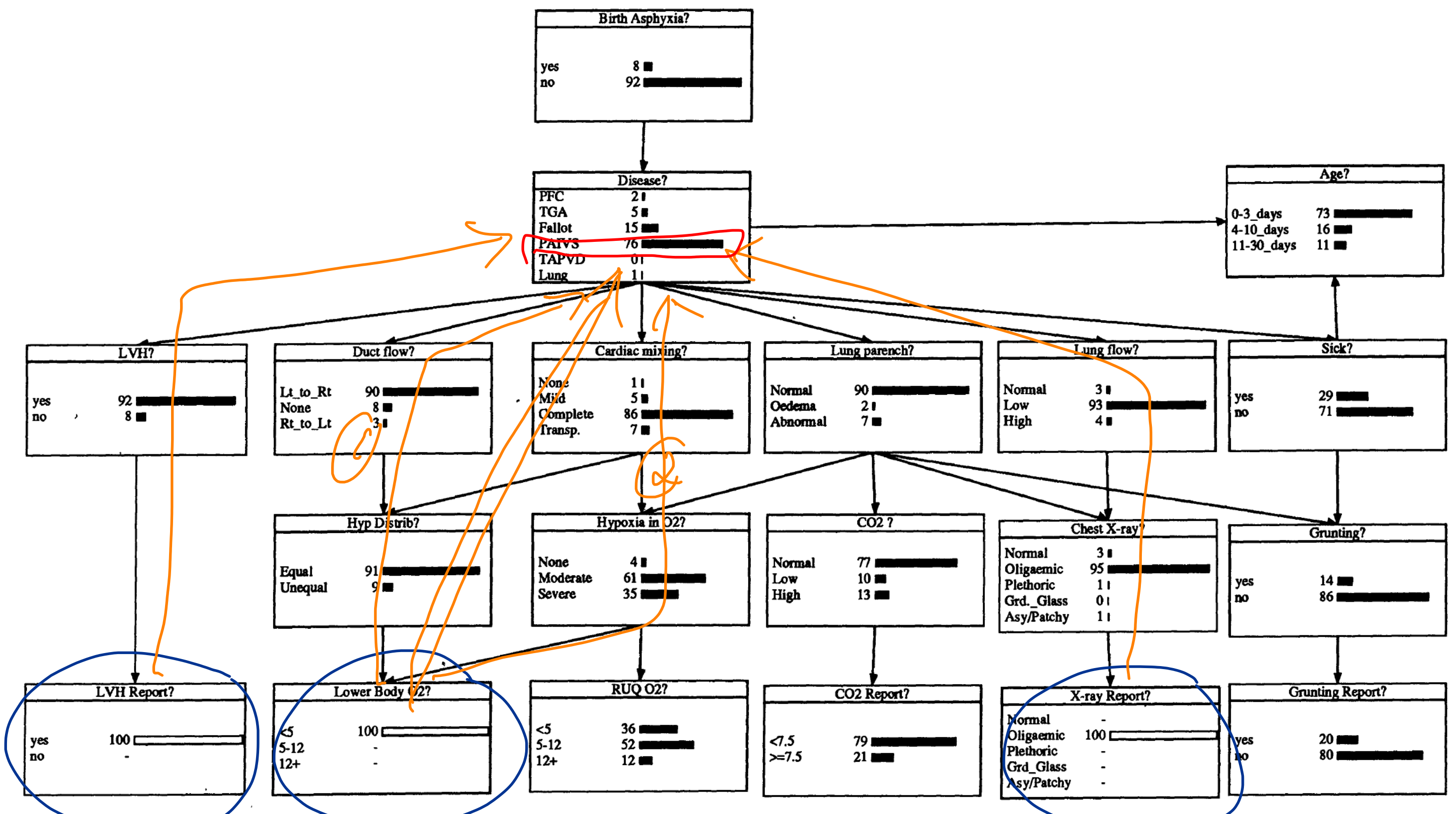


FIG. 4. *Status after propagation of additional evidence X-ray report = oligaemic and Lower body $O_2 < 5$.*

Moralize

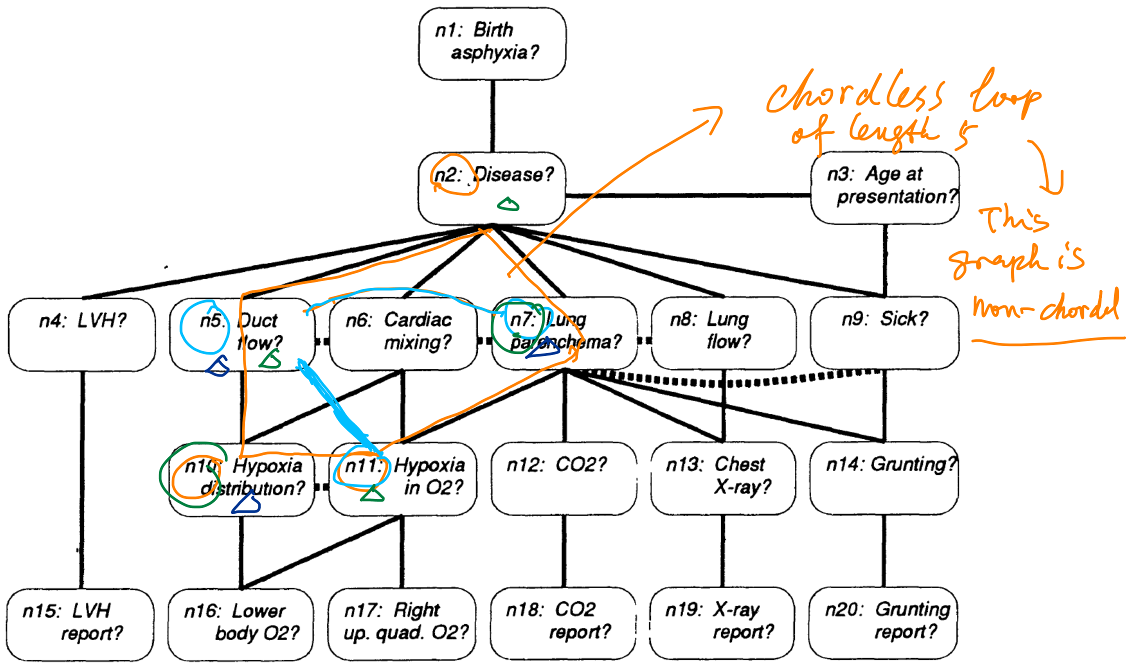


FIG. 9. *Moral graph formed from CHILD network by joining unconnected parents and dropping directions. The joint distribution of the variables is Markov with respect to this graph.*

Triangulation and Maximum Cardinality Search

chordal graph

⇓

junction tree representation

nonchordal.

$P(X_A)$

$= \sum_{X_B,X_C,X_D} P(X_A,X_B,X_C,X_D)$

$= \sum_{B,C,D} \psi_{AB}(X_A,X_B)\times\psi_{CD}(X_C,X_D)\,\psi_{BC}(X_C,X_B)\times\psi_{AD}(X_A,X_D)$

$\sum_{B,C}\psi_{AB}\psi_{BC}\sum_{D}\psi_{CD}\psi_{AD} \to$ fn. of A,C

labeled neighbors

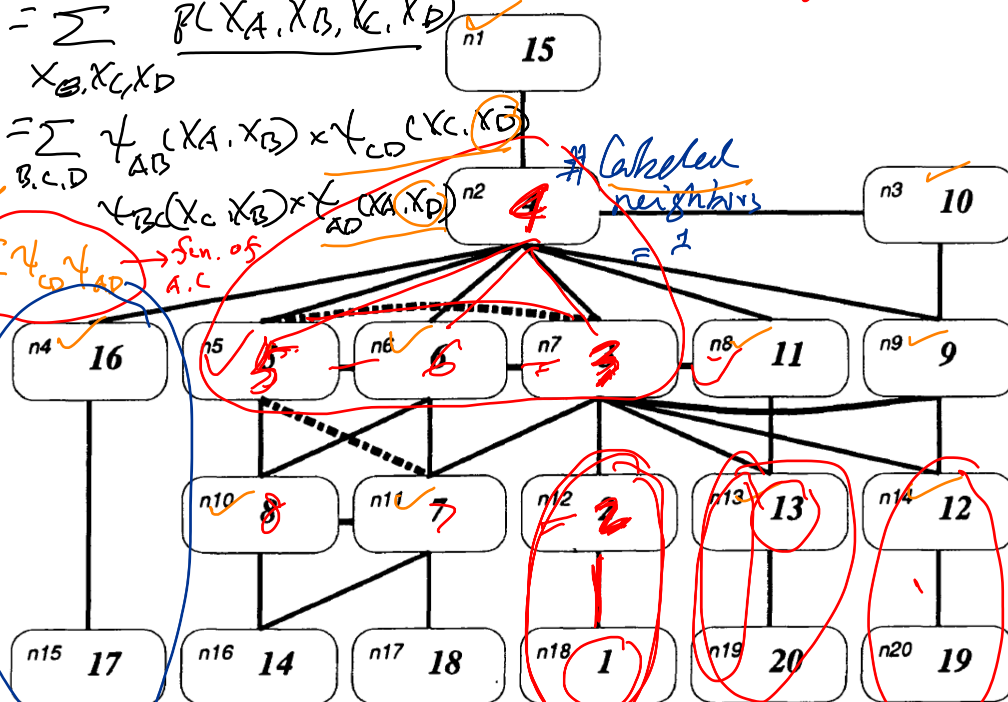
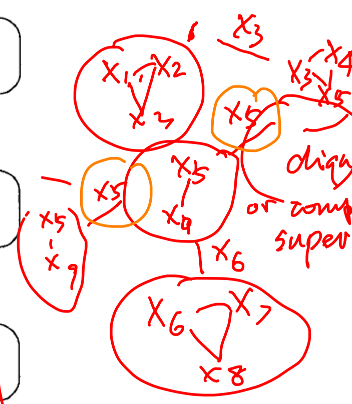


FIG. 11. *A perfect ordering of the nodes in CHILD arising from maximum cardinality search.*



clique or complete supernode

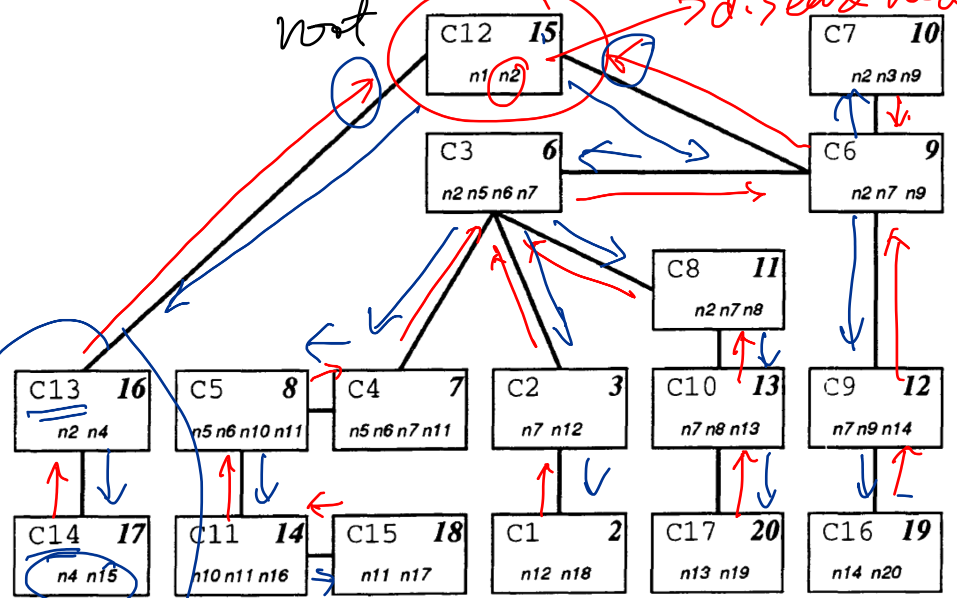
connected cliques

$\psi_v(X_v) = P(\text{LVH report} \mid \text{LVH status}) \cdot P(\text{LVH status})$

Construct Junction Tree and Define Potentials on Maximal Cliques

root

disease node.



1st pass (upstream pass)

$P(n_1, n_2)$

disease states

$S = C_{13} \cap C_{14}$

$= n_4$

$\psi_{C_{12}}(X_1, X_2)$

$\propto P(X_1, X_2)$

FIG. 12. *Junction tree of cliques derived from perfect ordering of the CHILD nodes. The members of each clique are shown, the highest label among the members is shown in the top right-hand corner, while the corresponding ordering of the cliques is shown in the top left-hand corner.*

$P(X_2) \propto \sum_{X_1} \psi_{C_{12}}(X_1, X_2)$

C_{13} : n_2 & n_4

C_{14} : n_4 & n_{15}

$m[C_{14} \to C_{13}]$

Introduce Evidence and Propagate Probabilities

V: source

W: target

S = separation set

$\underline{V \cap W}$

1) initialization

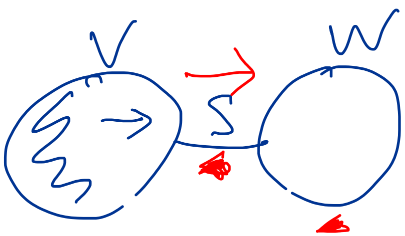
C_{13}

$\boxed{\Psi_V^{(0)}(X_V)}$

$\Psi_W^{(0)}(X_W)$

$\phi_S^{(0)}(X_S) = \underline{1}$

	(a)		(b)		(c)		(d)		(e)	
Disease	**LVH?**		**LVH?**		**LVH?**		**LVH?**		**LVH?**	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
PFC	0.100	0.900	0.090	0.045	0.004	0.002	0.004	0.002	0.015	0.007
TGA	0.100	0.900	0.090	0.045	0.031	0.016	0.031	0.016	0.109	0.055
Fallot	0.100	0.900	0.090	0.045	0.026	0.013	0.026	0.013	0.091	0.045
PAIVS	0.900	0.100	0.810	0.005	0.180	0.001	0.180	0.001	0.634	0.004
TAPVD	0.050	0.950	0.045	0.048	0.002	0.002	0.002	0.002	0.008	0.008
Lung	0.100	0.900	0.090	0.045	0.004	0.002	0.004	0.002	0.016	0.008



S

2) $\underline{\text{Propagation:}}$

	LVH? (a)		**LVH?** (b)		**LVH?** (c)		**LVH?** (d)		**LVH?** (e)	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
	1.000	1.000	0.900	0.050	0.900	0.050	0.248	0.036	0.872	0.128

$S \subset V$

$\sigma_{V \to W}(X_S) = \sum_{X_V \setminus X_S} \Psi_V^{(0)}(X_V)$

C_{14}

$\Psi_W^{(1)}(X_W) = \Psi_W^{(0)}(X_W) \times \dfrac{\sigma_{V \to W}(X_S)}{\phi_S^{(0)}(X_S)}$

LVH Report?	**LVH?** (a)		**LVH?** (b)		**LVH?** (c)		**LVH?** (d)		**LVH?** (e)	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Yes	0.900	0.050	0.900	0.050	0.900	0.050	0.248	0.036	0.872	0.128
No	0.100	0.950	0	0	0	0	0	0	0	0

FIG. 13. *Propagation of evidence through cliques C_{13} and C_{14} of junction tree:* (a) *initial potentials,* (b) *after incorporation of evidence LVH-report = yes,* (c) *after propagation through rest of network and back to C_{13},* (d) *final potentials,* (e) *marginal tables after normalisation.*

Final step $\phi_S^{(1)}(X_S) = \sum_{X_W \setminus X_S} \Psi_W^{(1)}(X_W)$

Comments:

1. Read Spiegelhalter, David J.; Dawid, A. Philip; Lauritzen, Steffen L.; Cowell, Robert G. Bayesian Analysis in Expert Systems. Statist. Sci. 8 (1993), no. 3, 219--247. doi: 10.1214/ss/1177010888. http://projecteuclid.org/euclid.ss/1177010888.

2. Convince yourself the numbers in Figure 13 are correct.

3. We will proceed to approximate inference.

4. Enjoy your Fall break!

5. Fill out the midterm survey: https://goo.gl/forms/AMfJ1t1d0gQbgQXI3