Lecture 5: Connections and Differences between Directed Acyclic and Undirected Graphical Models

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Lecture 4 Undirected Graphical Models: Main Points Again

Representation of Undirected Graphical Models

- Useful for describe correlations, especially when the directionality of causal influences is unclear or unrealistic.
- Gibbs distribution as a way to represent the joint probability distributions, with factors determining *affinity/interaction* among relevant variables
- Three ways of decreasing strength to read conditional independences from an UG: global, local and pairwise Markov properties.
  - Equivalent when the joint distribution is *positive* (counter-examples if without positivity).
  - For positive distributions, factorization and global Markov property are equivalent (Markov property to factorization established by Hammersley-Clifford-Besag theorem).
- Reading (optional but recommended): Chapter 7, *Gaussian Network Models*, Koller and Friedman (2009).

DAG to UG

Definition: The moral graph $\mathcal{M}(G)$ of a Bayesian network structure $\mathcal{G}$ over $\mathcal{X}$ is the undirected graph over $\mathcal{X}$ that contains an undirected edge between X and Y if: (a) there is a directed edge between them (in either direction), or (b) X and Y are both parents of the same node.

DAG to UG

Result: Let $\mathcal{G}$ be any Bayesian network graph. The moralized graph $\mathcal{M}(G)$ is a minimal I-map for $\mathcal{G}$. (Example on blackboard for moralization)

Question: when do we lose conditional independence after moralization? (v-structure)

Proposition: If the directed graph G is moral, then its moralized graph $\mathcal{M}(G)$ is a perfect map of $\mathcal{G}$.

UG to DAG: Example

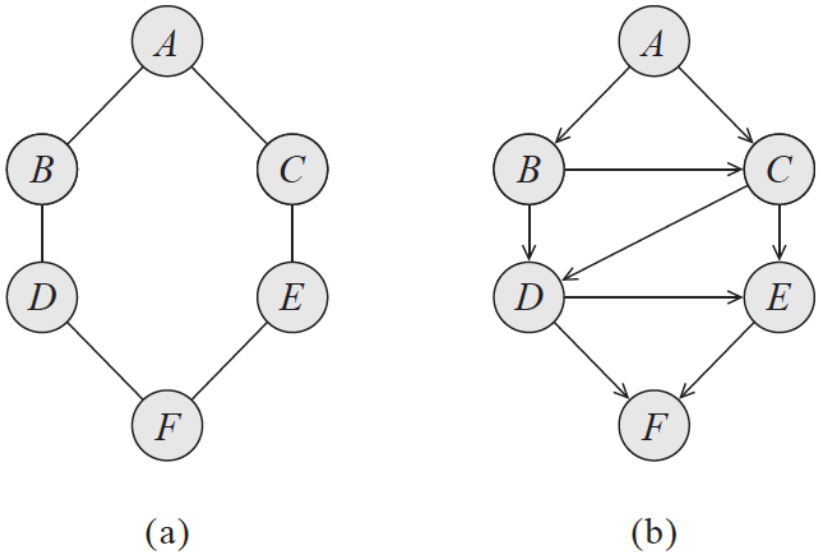


Figure 4.13 Minimal I-map Bayesian networks for a nonchordal Markov network. (a) A Markov network $\mathcal{H}_\ell$ with a loop. (b) A minimal I-map $\mathcal{G}_\ell$ Bayesian network for $\mathcal{H}$.

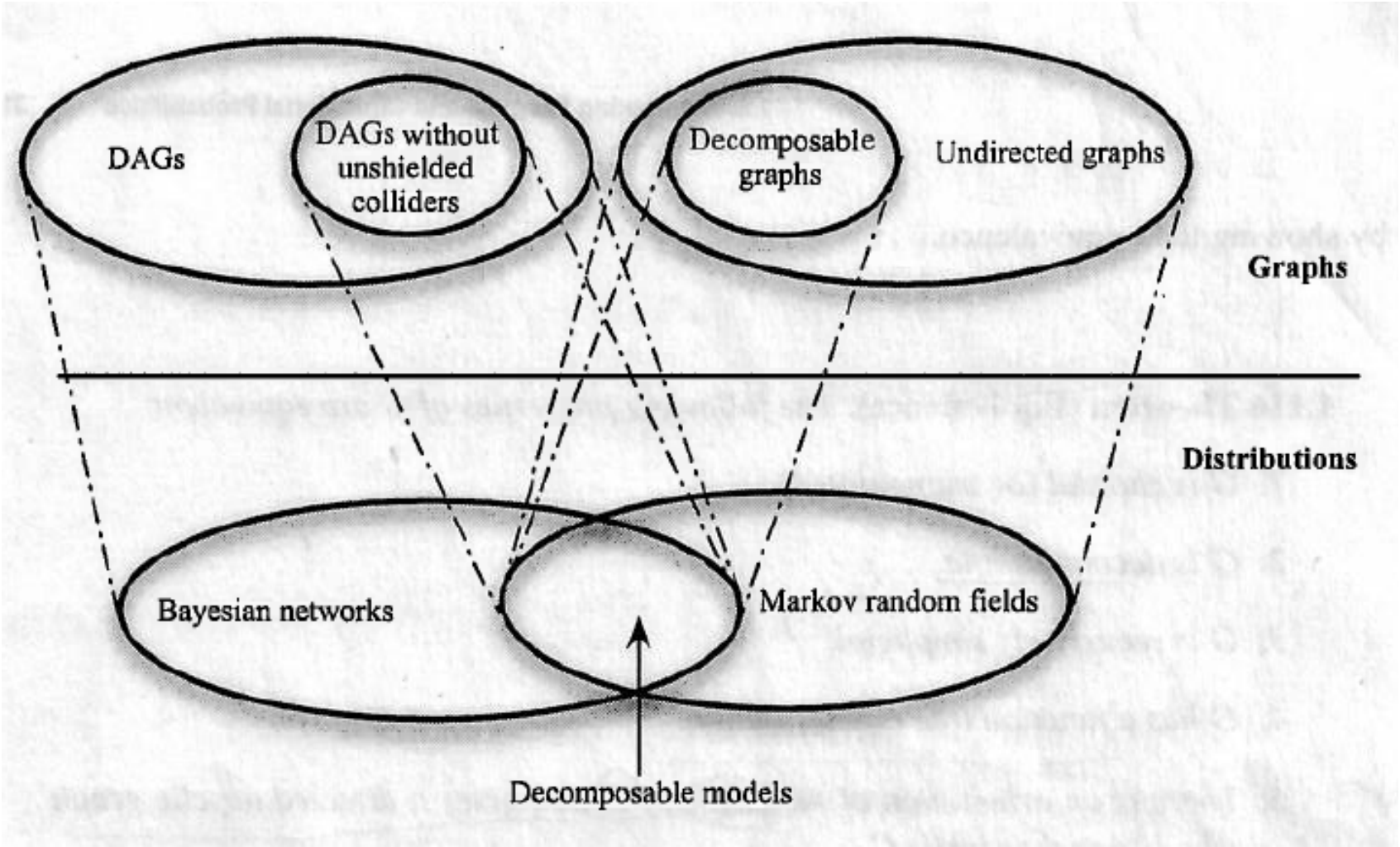
Theorem 4.10. Let $\mathcal{H}$ be a Markov network structure, and let $\mathcal{G}$ be any Bayesian network minimal I-map for $\mathcal{H}$. Then $\mathcal{G}$ can have no immoralities.

Chordal Graphs

Definition: A graph is **chordal** (also called triangulated) if it contains no chordless cycles of length greater than 3. Here, we say a cycle in G is *chordless* if all pairs of non-adjacent pairs in the cycles are not neighbors.

Theorem 4.13. Let $\mathcal{H}$ be a chordal Markov network. Then there is a Bayesian network $\mathcal{G}$ such that $\mathcal{I}(\mathcal{H}) = \mathcal{I}(\mathcal{G})$.

Venn Diagram for DAG and UG



Comment

- Next lecture: Other variants of graphical models. Log-linear model for multivariate discrete data in more detail.
- **Reading**:

required Lauritzen, S.L. and Spiegelhalter, D.J., 1988. Local computations with probabilities on graphical structures and their application to expert systems. Journal of the Royal Statistical Society. Series B (Methodological), pp.157-224. (To prepare for inference)

optional Chapter 7, Koller and Friedman (2009). Exponential family. Will review when needed.